Barclays Global Investors

The Opportunity Set

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(based on Grinold and Taylor, forthcoming *Journal of Portfolio Management*)

What is the Opportunity Set?

The ex post portfolio residual return may be decomposed as follows:

Portfolio Residual Return $= \theta_p = h'r$

$$= \frac{\alpha'\Omega^{-1}r}{2\lambda}$$

$$= \frac{\sqrt{r'\Omega^{-1}r}}{2\lambda}\frac{\alpha'\Omega^{-1}r}{\sqrt{r'\Omega^{-1}r}\sqrt{\alpha'\Omega^{-1}\alpha}}\sqrt{\alpha'\Omega^{-1}\alpha}$$

$$= \left[\frac{\sqrt{r'\Omega^{-1}r}}{2\lambda}\right]\left[\frac{\alpha'\Omega^{-1}r}{\sqrt{r'\Omega^{-1}r}\sqrt{\alpha'\Omega^{-1}\alpha}}\right]\left[2\lambda\sqrt{h'\Omega h}\right]$$

$$= \sqrt{r'\Omega^{-1}r}\left[\frac{\alpha'\Omega^{-1}r}{\sqrt{r'\Omega^{-1}r}\sqrt{\alpha'\Omega^{-1}\alpha}}\right]\sqrt{h'\Omega h}\ .$$

Hence:

$$\theta_p = OS \; x \; IC \; x \; Risk,$$

where

$$OS \equiv \sqrt{r'\Omega^{-1}r}$$

IC = realised information coefficient

$Risk$ = ex ante level of risk

Note that OS is a pure number (i.e. dimensionless).

The Opportunity Set as "Effective Breadth"

$$E(OS) = E\left[\sqrt{r'\Omega^{-1}r}\right] \approx \sqrt{E\left(r'\Omega^{-1}r\right)} = \sqrt{N}$$

Proof: Since, $E\left(r'\Omega^{-1}r\right)$ is a scalar,

$$E(OS^2) = E\left(r'\Omega^{-1}r\right) = trace\left[E\left(r'\Omega^{-1}r\right)\right]$$
$$= trace\left[E\left(\Omega^{-1}rr'\right)\right] = trace\left[\Omega^{-1}E(rr')\right]$$
$$= trace\left[\Omega^{-1}\Omega\right] = trace\left[I_N\right] = N.$$

Thus, $E(OS^2) = N$ and so $E(OS) \approx \sqrt{N}$ (because of Jensen's Inequality.)

The expected value of the opportunity set is approximately equal to the square root of the breadth of the portfolio

Intuition: if rr' is "large" relative to the covariance matrix Ω such that $OS \equiv \sqrt{r'\Omega^{-1}r} > \sqrt{N}$, then the market offered abnormal opportunities in the period for which *r* is calculated—tantamount to having more breadth. At the other extreme, suppose the market return vector was null, then *OS=0* and it is as if there were no breadth at all in the portfolio (i.e. no assets).

The Opportunity Set and the Fundamental Law of Active Management

We have:

$$\theta_p = OS \text{ x } IC \text{ x } Risk,$$

θ_p = *ex post* residual return

Risk = *ex ante* risk

But if

ex post risk ≈ ex ante risk

then

$\theta_p/(Risk)$ ≈ ex post IR

$IR \approx OS \text{ x } IC$

(So if $IC \approx 1/3$, $OS \approx 3$ (for ACM), $IR \approx 1.0$)

$$IR \approx OS \text{ x } IC$$

Take expectations conditional on realized *IC:*

$$E(IR|IC)=E(OS)xIC$$

or

$$E(IR|IC)= \sqrt{N} \text{ x } IC,$$

- a form of the Fundamental Law of Active Management

Intuition: while the opportunity set is a useful concept for assessing particular episodes, it is nevertheless true that, on average and in the last analysis, only two things count: skill and breadth.

The Distribution of the Opportunity Set

So far we made no distributional assumptions about returns

But suppose returns are normally distributed

then

$$OS^2 = r'\Omega^{-1}r \sim \chi^2$$

and so, once again

$$E(OS^2) = E\{\chi^2(N)\} = N$$

What does the distribution of *OS* look like in this case?

Monte Carlo Study

Calibrated on 10-currency portfolio with US-dollar as numéraire

N=9 (US dollar = numéraire)

Ω = estimated using using daily fx for the Australian dollar, Canadian dollar, Euro, yen, New Zealand dollar, Norwegian krone, Swedish krona, Swiss franc, sterling, January 1999 - March 2007.

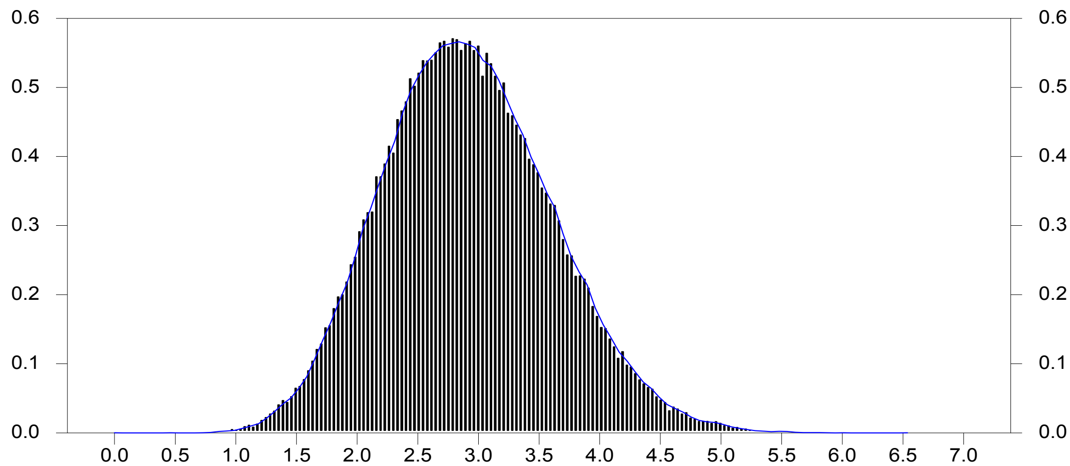
r drawn from multivariate normal distribution with covariance matrix Ω and OS calculated as

$$OS = \sqrt{\left(r'\Omega^{-1}r\right)}.$$

Repeated this for a total of 100,000 draws.

Results

E(OS)	**Median**	**5th Pctile**	**25th Pctile**	**75th Pctile**	**95th Pctile**	**Variance**
2.92	2.89	1.83	2.43	3.38	4.10	0.48



This looks suspiciously normal! ……..

Fisher (1922): if a random variable X is distributed as chi-square with N degrees of freedom then √(X) is approximately normally distributed with mean √[(2N-1)/2] and variance ½.

Note: For *N=9*, *√[(2N-1)/2]=2.92!*

What if the distribution of returns is fat-tailed?

Investigate through Monte Carlo experiments where *r* is assumed to be distributed as multivariate Student *t:*

$$r \sim t(\Omega, \nu).$$

ν determines how fat-tailed the distribution is

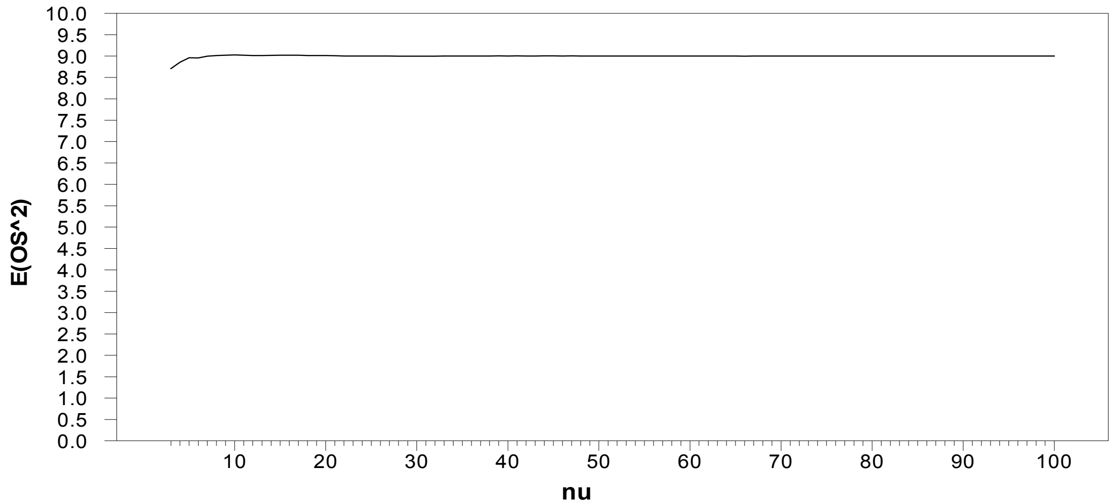
- for low values of ν, the distribution will have very fat tails

- as ν →∞, Student's *t* distribution becomes the normal distribution

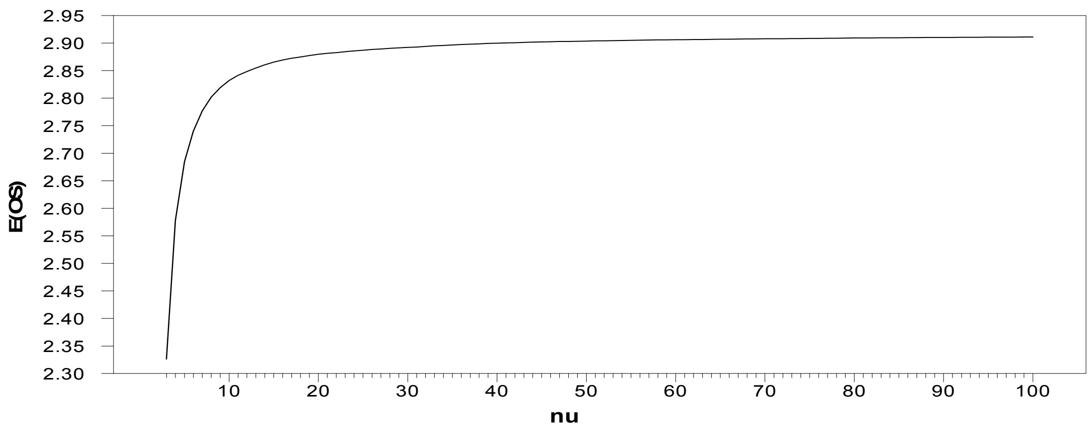
Again, Monte Carlo experimets calibrated on the ACM

100,000 draws for every value of ν from 3 to 100

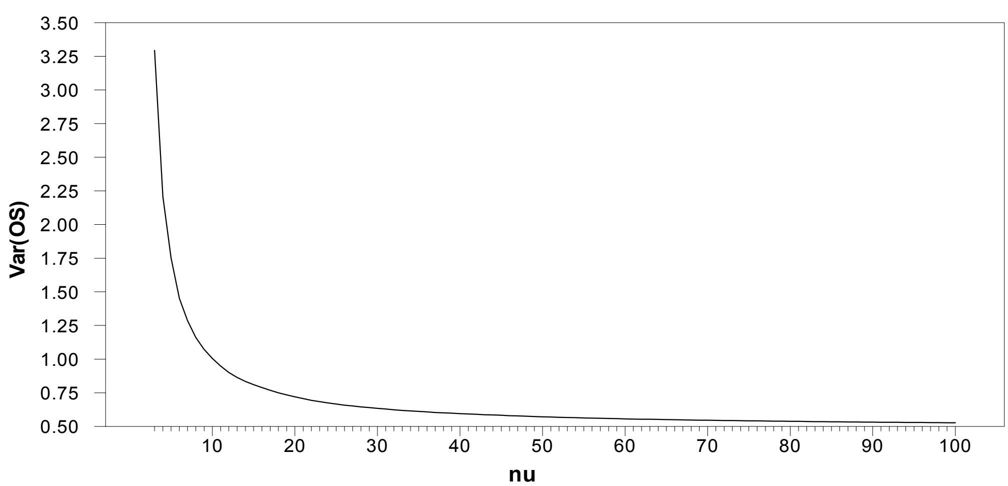
Mean Value of OS^2



Mean Value of *OS*



Variance of *OS*



Case Study: US Large Cap Market 1985-2007

-Monthly data, 276 observations

-Average # assets =1103

-BARRA risk model used to compute opportunity set

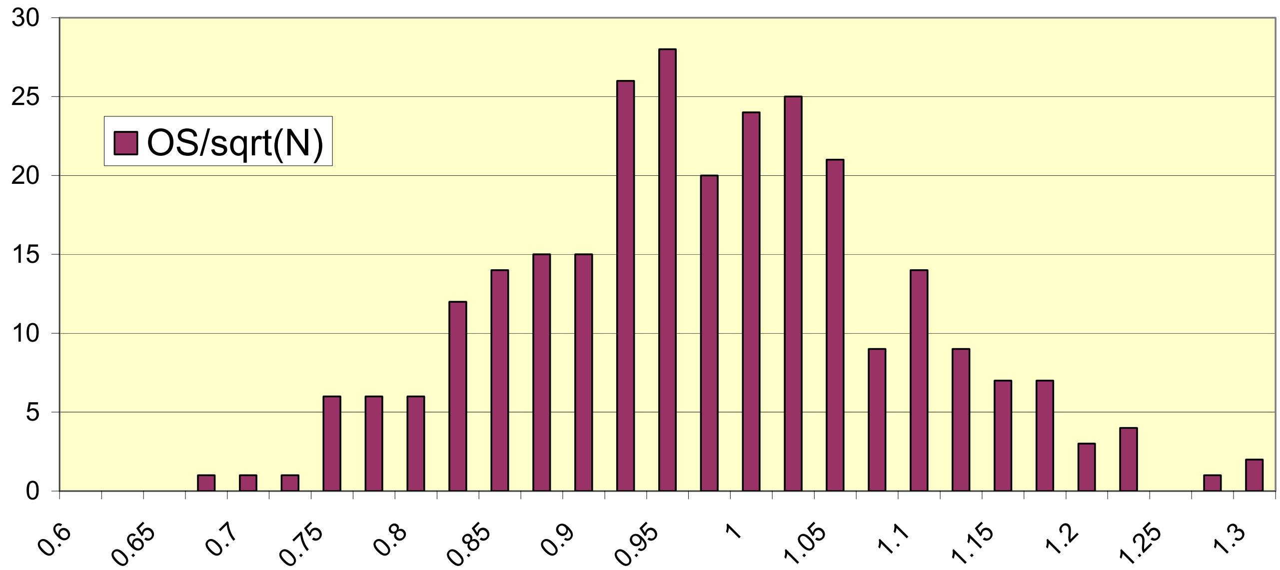


Exhibit 1: Scaled Opportunity Set Histogram, US Large Capitalization Market, 1985-2007

-Average = 0.96<1, as expected

-Maximum OS in September 1998

-Peaks in October 1987, but OS/$\sqrt{N}$ very close to one for that crash month, indicating there was no special opportunity to separate winners and losers; they were all losers.